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Diversity of rationality affects the evolution of cooperation

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By modifying the Fermi updating rule, we present the diversity of individual rationality to the evolutionary prisoner's dilemma game, and our results shows that this diversity heavily influences the evolution of cooperation. Cluster-forming mechanism of cooperators can either be highly enhanced or severely deteriorated by different distributions of rationality. Slight change in the rationality distribution may transfer the whole system from the global absorbing state of cooperators to that of defectors. Based on mean-field argument, quantitative analysis of the stability of cooperative clusters reveals the critical role played by agents with moderate degree values in the evolution of the whole system. The inspiration from our work may provide us a deeper comprehension toward some social phenomena.

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To understand the observed survival of cooperation among unrelated individuals in social communities when selfish actions provide a higher benefit [1,2], a lot of attention is being paid to the analysis of evolutionary dynamics of simple two-player games, such as the prisoner's dilemma game (PDG) [3–6]. In the standard form of PDG, each player may choose either to cooperate, C, or to defect, D, in any one encounter. If both players choose C, both get a payoff of R; if one defects while the other cooperates, D gets T, while C gets S; if both defect, both get P, where $T>R>P>S$. Presently, much interest has been given to evolutionary games in structured population [7–11], and heterogeneous scale-free topologies have been recognized as extremely potent promoters of cooperation [12]. Cooperation facilitating mechanisms are also proposed, including strategic complexity [13], direct and indirect reciprocitiy [14], asymmetriy of learning and teaching activities [15], individual similarity [16], random diffusion of agents on the grid [17], dynamic preferential selection [18], dynamic payoff matrices [19], fine-tuning of noise and uncertainties by strategy adoption [20], as well as the interplay between the evolution of cooperation and that of the interaction network [21].

This Rapid Communication concentrates on the diversity of an intrinsic property, individual rationality. Szabó's Fermi upgrading rule [22] has taken this vital and intrinsically determined property into account. During the evolution, a player, i, can follow the strategy of one of its randomly chosen neighbor, j, with the probability depending on their payoff difference (M_i-M_j),

$$W_{ij}=\frac{1}{1+\exp[(M_i-M_j)/T_i]}, \tag{1}$$

where T_i characterizes the level of rationality of agent i, which has also been viewed as stochastic noise [23] and related to coherence resonance [24]. $T_i=0$ denotes complete rationality, where the individual always adopts the best strategy determinately; while $T_i>0$, it introduces some irrational factor, that there is small possibility to select the worse one; $T_i\to\infty$ denotes that the individual is completely irrational, and its decision is random. Individual rationality values are the same for every game player in former works.

However, diversity plays an important role in the dynamics of complex systems, including social systems [25,26]. In real society, individual rationality is diversely distributed. Different individual rationality can result in different individual behavior. And local network topology largely depends on individual behavior [21]. Thus in social networks, an agent's connectivity may interrelate with its rationality. Within our study, diversity of rationality is introduced by the following function [27]:

$$T_i=NT_0\frac{k_i^{\beta}}{\sum_l k_l^{\beta}}, \tag{2}$$

where N is the total number of agents, T_0 is the average value of rationality, and k_i is the degree of agent i. A tunable parameter β determines the relationship between k_i and T_i. For fixed network topology, every particular value of β corresponds to a particular distribution of T_i.

By tuning the single parameter β, Eq. (2) allows to smoothly pass, first, from proportional to inversely proportional relationships between k_i and T_i, and second, from homogeneous to heterogeneous distributions of T_i [28]. Thus our model covers various cases, some of which might to some extent reflect the situations in real world. While $\beta<0$, agents with higher (lower) degree have lower (higher) values of rationality. $\beta>0$ displays the opposite situation. While $\beta=0$, rationality is uniformly (homogeneously) distributed. For player i, significantly high T_i could induce quite random behavior although (M_i-M_j) may be large. For example, when $\beta>0$, hubs, the minority in scale-free networks, may make quite irrational choices. This corresponds to a real case: there could exist a small number of irrational individuals in a very large population. On the contrary, very low T_i would heavily enhance agent's sensitivity toward higher payoff.

In our model, each node plays the classical PDG with all nodes connected. Self-interactions are excluded. The total

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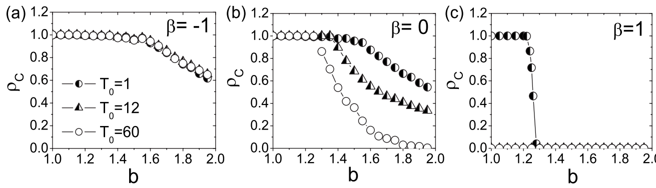

FIG. 1. The frequencies of cooperators ρ_c versus the temptation to defect b at different average rationality value $T_0=1.0$, 12.0, and 60.0 when (a) $\beta=-1$, (b) 0, and (c) 1.

payoff of an individual is the sum of the payoffs obtained in his two-player games with all other connected nodes. We choose $R=1$, $T=b>1$, and $S=P=0$ [29]. Here the Barabási-Albert (BA) scale-free network [30] is adopted since a plethora of biological and social real-world networks are mostly heterogeneous. It is built from a complete graph with $m_0=5$ nodes, the number of edges linked to the exiting nodes from the newly added node in each time step $m=2$ and the average degree $\bar{k}=4$. The total number of agents $N=20\,000$. Before the start of each game simulation, both strategies populate the scale-free network uniformly. We adopted a synchronous updating scheme. All the simulation results were obtained by averaging over 2000 generations after a transient time of 10 000 generations. Each data is obtained by averaging over 50 different network realizations with 20 runs for each realization.

Figure 1 shows the influence of rationality when $\beta=-1$, 0, and 1. As T_0 increases, the cooperation level of $\beta=0$ decreases evidently. But that of $\beta=-1$ has little variation, while that of $\beta=1$ drops much more sharply than the case of $\beta=0$. This means the well-known cooperation facilitation mechanism owe to hubs, as well as cooperative clusters [31] can be affected by the average rationality and the diversity of rationality. The robustness of cooperation is sensitive to T_0 and can either be greatly enhanced [Fig. 1(a)] or be severely weakened [Fig. 1(c)] by diversely distributed rationality.

Further information about how ρ_C is affected by different rationality distributions is provided by Fig. 2. It shows that ρ_C depends nonmonotonically on β, just as a gorge located in a plateau. Cooperation is effectively promoted on the plateau and seriously inhibited in the gorge. Near the gorge, surprisingly, slight distribution change can convert the whole system from global absorbing state of cooperators to that of defectors. It indicates that the evolution of cooperation is very sensitive to slight difference between the distributions of rationality. To highlight the intense variation in ρ_C, we call the gorge: *cooperation crisis*.

In order to examine how the extent of the heterogeneity of network topology affects *cooperation crisis*, we have made use of the model developed in Ref. [32], which allows to smoothly pass from a BA network ($\alpha=0$) to a random graph of the sort of Erdős-Rényi (ER) networks ($\alpha=1$) by tuning a single parameter α. Showed as Figs. 2(c) and 2(d), the gorge still exists, which to some extent indicates the universality of *cooperation crisis*. The plateau becomes lower and the gorge becomes wider in the case that b and α get larger, for cooperation is not highly promoted under homogeneous network topology.

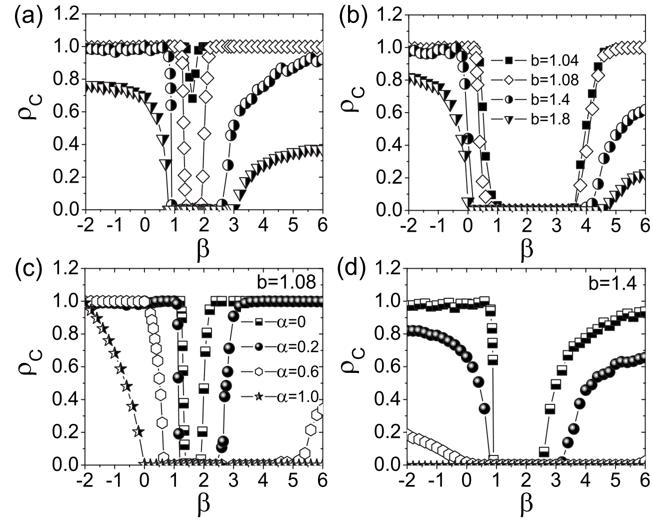

FIG. 2. The frequencies of cooperators ρ_C versus β. The average values of rationality [(a), (c), and (d)] $T_0=1.0$ and (b) 60.0.

To explain the main features of our findings, especially the *cooperation crisis*, we hereafter scrutinize in depth the microscopic evolution of cooperation. When $\beta=0$, heterogeneous spatial structure enable the cooperators to form stable clusters (C clusters). When $\beta\neq 0$, the stability of C clusters is to be analyzed.

While invaded by defectors, the local structure of a C cluster can be viewed as a C strategy hub surrounded by a number of periphery neighbors, most of which are cooperators. Two crucial transient processes corrupt C clusters: process (A): the hub cooperator of a C cluster adopts the strategy of a periphery defector and then transits to D strategy; process (D): a periphery cooperator adopts the strategy of the hub defector and then transits to D strategy. Another two transient processes consolidate C clusters: process (B): a hub

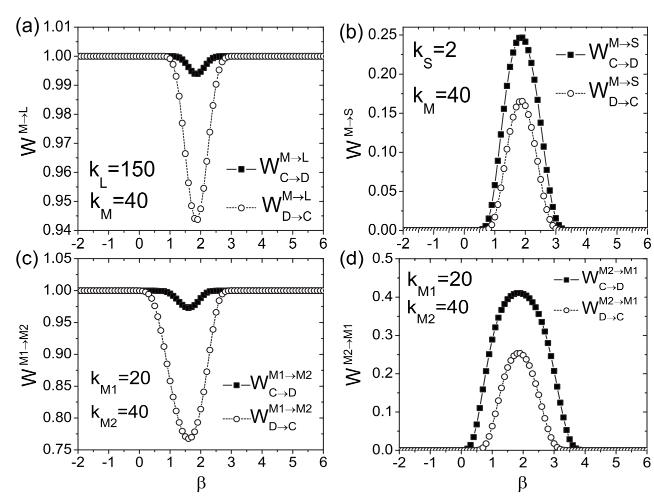

FIG. 3. The probability of AMDs' strategies transiting to the strategies of (a) ALDs, (b) ASDs, and [(c) and (d)] AMDs when $T_0=1.0$ and $b=1.4$. The solid squares and empty circles respectively denote the probabilities of processes (A) and (B) in graphs (a) and (c), as well as probabilities of processes (D) and (C) in graphs (b) and (d).

defector adopts the strategy of a periphery cooperator and then transits to C strategy; process (C): a periphery defector adopts the strategy of the hub cooperator of a C cluster and then transits to C strategy. For a hub, both processes (A) and (B) could happen; for a periphery agent, both processes (C) and (D) could happen. According to Eq. (1), four kinds of strategy transition probability correspond to these four dynamic processes. Based on mean-field approximation, imaging a localized block in the network, agent i is surrounded by k_i neighbors among which the cooperators have a fraction μ. The payoff difference between cooperator i and defector j (or defector i and cooperator j) can be denoted by $\mu(k_i-k_jb)$ [or $\mu(k_ib-k_j)$]. As the mean-field approximation is not always fit for the evolutionary games on networks, the following analysis can only be qualitative. From Eqs. (1) and (2), we gain the following four kinds of transition probability:

$$\begin{cases} \text{Process(A)}: & W_{C\to D}^{H\to P} = \dfrac{1}{1+\exp\left[\dfrac{\mu(k_H-k_Pb)}{T_0}(k^\beta/k_H^\beta)\right]} \\ \text{Process(D)}: & W_{C\to D}^{P\to H} = \dfrac{1}{1+\exp\left[\dfrac{\mu(k_P-k_Hb)}{T_0}(k^\beta/k_P^\beta)\right]} \\ \text{Process(B)}: & W_{D\to C}^{H\to P} = \dfrac{1}{1+\exp\left[\dfrac{\mu(k_Hb-k_P)}{T_0}(k^\beta/k_H^\beta)\right]} \\ \text{Process(C)}: & W_{D\to C}^{P\to H} = \dfrac{1}{1+\exp\left[\dfrac{\mu(k_Pb-k_H)}{T_0}(k^\beta/k_P^\beta)\right]}, \end{cases} \quad (3)$$

where the upper scripts H and P denote hub and periphery node, respectively. The term $(\bar{k}^\beta/k^\beta)$ remodifies and extends the Fermi rule. The strategy transition probability in a certain process can be regarded as the occurrence rate of this process. C clusters are unstable if the occurrence rates of process (A) [or (D)] are higher than that of (B) [or (C)].

To calculate the four kinds of transition probability, agents are approximately divided into three classes according to their connectivity (similar methods see Ref. [33]): (i) Agents with small values of degree (ASDs): $m \le k_S \le \bar{k}$; (ii) Agents with moderate values of degree (AMDs): $\bar{k} < k_M < k'$; (iii) Agents with large values of degree (ALDs): $k' \le k_L \le k_{\max}$ (k''s value depends on system size). Figure 3 shows the strategy transition probabilities of AMDs (the upper scripts S, M, and L respectively denote ASD, AMD, and ALD). Sharp variation can be observed at the region of *cooperation crisis* (see Fig. 2), where process (A) [or (D)] always has larger occurrence rate than process (B) [or (C)]. These indicate the advantage of the D strategy when one of the two game participators is a AMD. Hence C clusters lose their stability. In our calculation, the BA network has the largest degree $k_{\max}=419$ and the smallest degree $k_S=m=2$. The above results are qualitatively robust to the values of k_S, k_M, k_L, k', and μ.

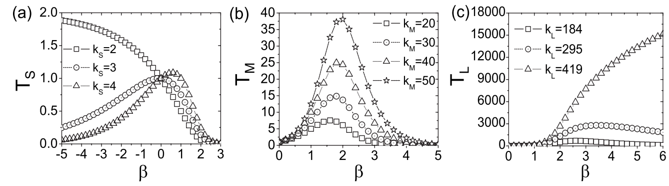

FIG. 4. The rationalities of different degree agents T_S, T_M, and T_L versus β when $T_0=1.0$.

As an example, here we set the degree of ALDs: $k_L=150$; the degree of AMDs: $k_{M1}=20$ and $k_{M2}=40$; the degree of ASDs: $k_S=2$; $k'=70$ and $\mu=0.75$.

Figure 4 shows how the rationality values depend on β for the three classes of agents. The T values of the agents with smallest (largest) degree monotonously decrease (increase), while that of agents with other degree values varies in a nonmonotonous fashion. We should note that at the region of *crisis*, peak values of T_M are reached. These peaks nicely explain the large variation in Fig. 3. At this region, irrational defective AMDs and ALDs are less likely to be affected by periphery cooperative AMDs and ASDs; irrational periphery defective AMDs are less likely to be affected by cooperative AMDs and ALDs. Thereby cluster-forming mechanism of cooperators loses its efficiency and *cooperation crisis* occurs. Compared with the case without diversity of rationality, connected hubs no longer help to form compact C clusters and promote cooperation unless they are sufficiently rational to pursue higher payoff.

Cluster-forming mechanism of cooperators is enhanced outside of cooperation crisis. On the left side of crisis (see Fig. 2), especially when $\beta<0$, ALDs and AMDs are extremely rational. Compared with when $\beta=0$, it is much easier for periphery cooperative ASDs to overturn the defective ALDs and AMDs, which would then help to form compact C cores. On the right side of crisis, ASDs and AMDs are rational, but the behavior of ALDs become random, and their strategy transition probabilities only depend on the proportion of cooperators and defectors connected to them. Once cooperative ASDs and AMDs take over a defective ALD, its high payoff would transit nearly all its periphery agents to cooperators. Then the chance of returning to defective strategy is greatly reduced. These mechanisms promote cooperation due to the heterogeneity of the rationality distributions, which cannot be affected by T_0. Therefore, the robustness of cooperation is enhanced even if T_0 is significantly high (see Figs. 1 and 2). Notably, while most works emphasize the role of hubs (ALDs) for promotion of cooperation [12,26,31], here we find that rational AMDs are of great importance.

To sum up, by modifying the Fermi updating rule, we introduce the diversity of individual rationality to evolutionary games, and our results reveal that this diversity heavily influences the emergence of cooperation. Cluster-forming mechanism of cooperators can either be highly enhanced or severely deteriorated by different distributions of rationality (even slightly different).

The crucial contribution made by AMDs may provide some sociological inspiration. If we could analogize agents' degree to certain social rank, then AMDs might be related to the middle class, which could serve as a social stabilizer, pointed out by Huntington [34]. However, middle class can also play subversive role, argued by Huntington's opponents. Society is an open complex system; then social properties of individuals and their distributions could be influenced by various factors, such as economical or political. Probably, since the organization of society largely depends on the emergence of cooperation [35–37], such two contrary effects could be relevant to the various rationality modes of middle class, which correspond to different sorts of rationality distributions as have been shown in our work. Further investigation on the diversity of rationality under various network topologies, as well as the coevolution with the game dynamics, might yield new insights toward complex social phenomena.

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